## COMP 122/L

Summer 2023

## Working with Truth Tables, Boolean Formulas, and Circuit Diagrams (Answers)

1.) In two's complement addition, the value of the overflow bit can be determined by looking at the leftmost bit of the two input operands, as well as the leftmost bit of the output. Specifically, if the two input operands have the same leftmost bit, but the output has a different leftmost bit, it means the sign changed, so overflow occurred.
1.a.) Complete the truth table below reflecting this information. This truth table has the following inputs and outputs:

- Input: A (leftmost bit of the first operand)
- Input: B (leftmost bit of the second operand)
- Input: R (leftmost bit of the result)
- Output: V (set to 1 if overflow occurred, else 0 )

| $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{R}$ |  |
| ---: | ---: | ---: | ---: |
| 0 | 0 | 0 | $\mathbf{V}$ |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |
|  |  | 0 |  |

1.b.) Write this same information below as a Boolean formula in sum-of-products notation.

```
V = !A!BR + AB!R
```

2.) Consider the Boolean formula below, which is written in sum-of-products notation:
$A+B!C$

Write out a truth table below which is equivalent to this formula.

| A | B |  | Output |
| ---: | ---: | ---: | ---: |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

3.) Consider the following Boolean formula: ! (AB) + ! (CD)
3.a.) Using De Morgan's laws, rewrite this formula into sum-of-products notation.
$!A+!B+!C+!D$
3.b.) Write out a truth table below which is equivalent to this formula.

| A | B | C | D | Output |
| ---: | ---: | ---: | ---: | ---: |
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 |
|  |  |  | 1 | 1 |

4.) Consider the following Boolean formula: $A$ ! $B+C D$

Write out an equivalent circuit below.

5.) Consider the following Boolean formula: ! $\mathrm{A}+$ ! BC

Write out an equivalent circuit below.

6.) Consider the following circuit:


Write an equivalent Boolean formula representing this circuit.
$X=A B+C D$
7.) Consider the following circuit:


Write an equivalent Boolean formula representing this circuit.
$X=(A+B)(!(C D))$

